Additional problems for TI-ADSI, Q3 2014

Chapter 5: Transform analysis of LTI systems

Problem 1. Group delay in FIR filters

Consider a FIR filter with a single zero located at $z_0 = re^{j\theta}$. This means that

$$\begin{aligned} H(\omega) &= 1 - re^{j\theta}e^{-j\omega} \\ &= 1 - r\cos(\omega - \theta) + jr\sin(\omega - \theta) \end{aligned}$$

1. Show that the phase response is

$$\Psi(\omega) = \tan^{-1}\left(\frac{r\sin(\omega - \theta)}{1 - r\cos(\omega - \theta)}\right)$$

2. Show that the group delay becomes

$$\tau(\omega) = \frac{r^2 - r\cos(\omega - \theta)}{1 + r^2 - 2r\cos(\omega - \theta)}$$

3. What physical unit is used to measure group delay ?

Assume that the signal in a 48 kHz audio system is filtered with a FIR filter with the impulse response $h(n) = \{1,\ \frac{1}{2}\}$.

4. Use the results from the first part of the problem to plot the group delay, i units of μs, in the filter as a function of the frequency measured in Hz. Matlab's `grpdelay` command can be used to verify your results.

Mathematical hints for the problem:

$$\cos^2(x) + \sin^2(x) = 1, \qquad \frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx} \qquad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{df(x)}{dx}g(x) - f(x)\frac{dg(x)}{dx}}{g^2(x)}$$

Problem 2. All-Pass filters

An All-Pass filter has the following system function

$$H(z) = \frac{0.57 + 0.23z^{-1} + z^{-2}}{1 + 0.23z^{-1} + 0.57z^{-2}}$$

1. First, without performing calculations, try to predict what the impulse response of the filter will look like. Second, plot the impulse response and compare with your prediction.

Next, assume that the filter is to be implemented on a fixed point 16 bits signal processor, e.g. an Analog Devices Blackfin. This implies that the coefficients must be quantized to (1.15) format.

2. Is the filter still an all-pass filter when the coefficients have been quantized ?

3. If you recall how the calculation works, quantize the filter coefficients to 16 bits in (1.15) format.

Problem 3. Allpass filters and reverberators

In this problem the goal is to try out how two different reverberation algorithms sounds in practice.

1. Read example 5.7 and 5.9. In particular, pay attention to the two impulse responses (Fig. 5.26 and 5.32) and compare them.

2. Find a way of implementing the reverberators in Matlab.

3. Assume we will use the reverberators on a 48 kHz system. What are suitable values of a and D?

4. On campusnet you can find a short piece of music sampled at 48 kHz. Use `wavread` to load this into Matlab and play it with `audioplayer`.

5. Run the music through the two reverberators and listen to the output. Can you tell the difference?

Problem 4. Filter decomposition

Let the system function for a FIR filter be given by

$$H(z) = 1 - 3z^{-1} + \frac{5}{2}z^{-2} - z^{-3}$$

1. Decompose the system function into a product of a minimum-phase filter and an all-pass filter, $H(z) = H_{min}(z)H_{ap}(z)$.
2. Demonstrate that $H(z)$ and $H_{min}(z)$ have the same amplitude response.
3. Repeat for the following filter

$$H(z) = \frac{1 + \frac{3}{2}z^{-1} - 9z^{-2} + 4z^{-3}}{1 - \frac{1}{2}z^{-1}}$$

4. What are the real world applications of this kind of filter decomposition ?

Problem 5. Filter decomposition

Consider a FIR filter with the following difference equation

$$y(n) = x(n) + 2x(n-1) + 2x(n-2)$$

1. Show that the FIR filter is not minimum-phase.
2. Find the difference equation for the corresponding minimum-phase FIR filter.